

Structural Equation Modeling



## Today's goal:

Teach how to test theoretical models with SEM.

## Outline:

- Running, trimming, and evaluating saturated models
- Expanding the model to include additional variables



## **Testing full models** The real power of SEM



Step 1: Build your CFA  $\checkmark$ 

Step 2: Analyze the marginal effects of the manipulations  $\checkmark$ 

Step 3: Set up a model based on theory  $\checkmark$ 

Step 4: Test and trim a saturated version of this model







## Steps:

- Trim the model
- Get model fit statistics
- Optional: expand the model
- Reporting



```
In R:
```

```
model <- 'satisf =~ s1+s2+s3+s4+s5+s6+s7
quality =~ q1+q2+q3+q4+q5+q6
control =~ c1+c2+c3+c4
underst =~ u2+u4+u5
satisf ~ quality+control+underst+citem+cfriend+cgraph+cig+cfg
quality ~ control+underst+citem+cfriend+cgraph+cig+cfg
control ~ underst+citem+cfriend+cgraph+cig+cfg
underst ~ citem+cfriend+cgraph+cig+cfg';</pre>
```

fit <- sem(model,data=twq,ordered=names(twq[9:31]),std.lv=TRUE);</pre>

```
summary(fit);
```



Rules:

- Start with the least significant and least interesting effects (those that were added for saturation)
- Work iteratively
- Manipulations with >2 conditions: remove all dummies at once (if one is significant, keep the others as well)
- Interaction+main effects: never remove main effect before the interaction effect (if the interaction is significant, keep the main effect regardless)



	Estimate	Std.err	Z-value	P(> z )
<pre>(factors)</pre>				
Regressions:				
satisf ~				
quality	0.439	0.076	5.753	0.000
control	-0.838	0.107	-7.804	0.000
underst	0.090	0.073	1.229	0.219
citem	0.318	0.265	1.198	0.231
cfriend	0.014	0.257	0.054	0.957
cgraph	0.308	0.229	1.346	0.178
cig	-0.386	0.356	-1.082	0.279
cfg	-0.394	0.357	-1.103	0.270
quality ~				
control	-0.764	0.086	-8.899	0.000
underst	0.044	0.073	0.595	0.552
citem	0.046	0.204	0.226	0.821
cfriend	0.165	0.251	0.659	0.510
cgraph	0.009	0.236	0.038	0.970
cig	0.106	0.317	0.334	0.738
cfg	0.179	0.374	0.478	0.632



-0.308	0.066	-4.695	0.000
0.053	0.240	0.220	0.826
0.009	0.221	0.038	0.969
-0.043	0.239	-0.181	0.857
-0.148	0.341	-0.434	0.664
-0.273	0.331	-0.824	0.410
0.367	0.220	1.666	0.096
0.534	0.217	2.465	0.014
0.556	0.227	2.451	0.014
-0.106	0.326	-0.324	0.746
-0.178	0.320	-0 <b>.</b> 555	0.579
	-0.308 0.053 0.009 -0.043 -0.148 -0.273 0.367 0.534 0.556 -0.106 -0.178	-0.308 $0.066$ $0.053$ $0.240$ $0.009$ $0.221$ $-0.043$ $0.239$ $-0.148$ $0.341$ $-0.273$ $0.331$ $0.367$ $0.220$ $0.534$ $0.217$ $0.556$ $0.227$ $-0.106$ $0.326$ $-0.178$ $0.320$	-0.308 $0.066$ $-4.695$ $0.053$ $0.240$ $0.220$ $0.009$ $0.221$ $0.038$ $-0.043$ $0.239$ $-0.181$ $-0.148$ $0.341$ $-0.434$ $-0.273$ $0.331$ $-0.824$ $0.367$ $0.220$ $1.666$ $0.534$ $0.217$ $2.465$ $0.556$ $0.227$ $2.451$ $-0.106$ $0.326$ $-0.324$ $-0.178$ $0.320$ $-0.555$



Remove interactions -> (1) understandability, (2) quality, (3) control, and (4) satisfaction

Remove cgraph -> (1) satisfaction, and (2) quality



Remove citem and cfriend -> control

But wait... did we not hypothesize that effect?

Yes, but we still have citem+cfriend -> underst -> control!

In other words: the effect of item and friend control on perceived control is mediated by understandability!

Argument: "Controlling items/friends gives me a better understanding of how the system works, so in turn I feel more in control"



Remove citem and cfriend -> satisfaction

Remove understandability -> recommendation quality

- We hypothesized this effect, but it is still mediated by control.
- Argument: "Understanding the recommendations gives me a feeling of control, which in turn makes me like the recommendations better."

Remove understandability -> satisfaction

Same thing



Remove citem and cfriend -> recommendation quality

Remove cgraph -> control

Again: still mediated by understandability

Stop! All remaining effects are significant!



model <- 'satisf =~ s1+s2+s3+s4+s5+s6+s7 quality =~ q1+q2+q3+q4+q5+q6 control =~ c1+c2+c3+c4 underst =~ u2+u4+u5 satisf ~ quality+control quality ~ control control ~ underst underst ~ citem+cfriend+cgraph'



	Estimate	Std.err	Z-value	P(> z )
(factors)				
Regressions:				
satisf ~				
quality	0.418	0.080	5.228	0.000
control	-0.887	0.120	-7.395	0.000
quality ~				
control	-0.779	0.084	-9.232	0.000
control ~				
underst	-0.371	0.067	-5.522	0.000
underst $\sim$				
citem	0.382	0.200	1.915	0.056
cfriend	0.559	0.195	2.861	0.004
cgraph	0.628	0.166	3.786	0.000







```
Get modification indices
```

```
mods <- modindices(fit)
mods <- mods[mods$mi > 3.84 & !is.na(mods$mi),]
mods[order(-mods$mi),]
```

	lhs	ор	rhs	mi	mi.scaled	ерс	sepc.lv	sepc.all	sepc.nox
1	control	$\sim$	satisf	28.794	23.984	0.578	0.912	0.912	0.912
2	control	$\sim \sim$	underst	22.270	18.550	0.352	0.304	0.304	0.304
3	s3	$\sim \sim$	s4	20.785	17.313	0.157	0.157	0.156	0.156
4	underst	=~	q5	15.201	12.662	-0.150	-0.162	-0.161	-0.161
5	s2	$\sim \sim$	s7	10.021	8.347	0.101	0.101	0.100	0.100
6	satisf	=~	с3	8.796	7.327	-0.169	-0.286	-0.284	-0.284
7	underst	=~	s6	8.049	6.705	0.109	0.117	0.117	0.117

No useful modification indices in the regression part of the model (only stuff we had left from the CFA)



Item and factor fit should not have changed much (please double-check the r-squares!)

#### Great model fit!

- Chi-Square value: 306.685, df: 223 (value/df = 1.38)
- CFI: 0.994, TLI: 0.993
- RMSEA: 0.037 (great), 90% CI: [0.026, 0.047]



Satisfaction: 0.654

Perceived Recommendation Quality: 0.416

Perceived Control: 0.156

Understandability: 0.151

These are all quite okay



```
In model definition:
```

underst ~ cgraph+p1\*citem+p2\*cfriend

Then run:
 lavTestWald(fit,'p1==0;p2==0');

```
Result: Omnibus effect of control is significant (this is a chi-
square test)
$stat
[1] 8.386272
$df
```

```
[1] 2
```

```
$p.value
[1] 0.01509886
```







We subjected the 4 factors and the experimental conditions to structural equation modeling, which simultaneously fits the factor measurement model and the structural relations between factors and other variables. The model has a good\* model fit: chi-square(223) = 306.685, p = .0002; RMSEA = 0.037, 90% Cl: [0.026, 0.047], CFI = 0.994, TLI = 0.993.

\* A model should not have a non-significant chi-square (p > .05), but this statistic is often regarded as too sensitive. Hu and Bentler propose cut-off values for other fit indices to be: CFI > .96, TLI > .95, and RMSEA < .05, with the upper bound of its 90% CI below 0.10.



The model shows that the inspectability and control manipulations each have an independent positive effect on the understandability of the system: the full graph condition is more understandable than the list only condition, and the item control and friend control conditions are more understandable than the no control condition. Understandability is in turn related to users' perception of control, which is in turn related to the perceived quality of the recommendations. The perceived control and the perceived recommendation quality finally determine participants' satisfaction with the system.



```
model <- 'satisf =~ s_{1+s_{2+s_{3+s_{4+s_{5+s_{6+s_{7}}}}}}
             quality = \sim q1+q2+q3+q4+q5+q6
             control = \sim c1+c2+c3+c4
             underst = \sim u2+u4+u5
             satisf ~ pqs*quality+pcs*control
             quality ~ pcq*control
             control ~ puc*underst
             underst ~ piu*citem+pfu*cfriend+pgu*cgraph
             item2sat := piu*puc*(pcs+pcq*pqs)
             friend2sat := pfu*puc*(pcs+pcq*pqs)
             graph2sat := pgu*puc*(pcs+pcq*pqs)
             item2qual := piu*puc*pcq
             friend2qual := pfu*puc*pcq
             graph2qual := pgu*puc*pcq
             item2ctrl := piu*puc
             friend2ctrl := pfu*puc
             graph2ctrl := pgu*puc'
```











Error bars are smaller because total effects are mediated (mediation increases the accuracy of estimation)

- Values may be different because total effects are **modeled** (there may be some model misspecification)
- Which one should I use?
  - Marginal effect graphs are more "honest"
  - Use the p-values of the total effects (if needed); show the graphs of the marginal effects



# Expanding the model Add other variables



Expanding the model by adding additional variables

- Behavioral variables
- Personal and situational characteristics

Redo model tests and additional stats



We also measured:

- Recommendation inspection time (in seconds)
- Number of known recommendations (out of 10)
- Average rating (of the 10 recommendations)
- To add these, we need to revisit the factor model! Q: Why didn't we add these to begin with? A: Because they are less important!



#### Inspection time

Turn into minutes (large numbers throw off the algorithm) May be influenced by experimental conditions May influence understandability, control, etc.



twq\$mins <- twq\$time/60;</pre>

```
model <- 'satisf =~ s1+s2+s3+s4+s5+s6+s7
    quality =~ q1+q2+q3+q4+q5+q6
    control =~ c1+c2+c3+c4
    underst =~ u2+u4+u5
    satisf ~ quality+control+mins
    quality ~ control+mins
    control ~ underst+mins
    underst ~ mins+citem+cfriend+cgraph
    mins ~ citem+cfriend+cgraph'</pre>
```



Regressions:				
satisf ~				
quality	0.419	0.080	5.214	0.000
control	-0.897	0.122	-7.369	0.000
mins	0.258	0.102	2.534	0.011
quality ~				
control	-0.780	0.084	-9.258	0.000
mins	0.071	0.084	0.844	0.399
control ~				
underst	-0.367	0.067	-5.447	0.000
mins	0.129	0.090	1.434	0.152
underst $\sim$				
mins	0.245	0.088	2.793	0.005
citem	0.427	0.202	2.114	0.035
cfriend	0.660	0.201	3.288	0.001
cgraph	0.569	0.167	3.401	0.001
mins $\sim$				
citem	-0.165	0.098	-1.685	0.092
cfriend	-0.393	0.125	-3.156	0.002
cgraph	0.286	0.091	3.128	0.002



Number of known recommendations (out of 10)

- May be influenced by inspectability (remember the homework?)
- May influence understandability, control, recommendation quality, satisfaction



```
model <- 'satisf =~ s1+s2+s3+s4+s5+s6+s7
    quality =~ q1+q2+q3+q4+q5+q6
    control =~ c1+c2+c3+c4
    underst =~ u2+u4+u5
    satisf ~ quality+control+known
    quality ~ control+known
    control ~ underst+known
    underst ~ known+mins+citem+cfriend+cgraph
    mins ~ known+citem+cfriend+cgraph
    known ~ citem+cfriend+cgraph'</pre>
```



Regressions:				
satisf ~				
quality	0.462	0.083	5.561	0.000
control	-0.884	0.119	-7.447	0.000
known	-0.142	0.051	-2.778	0.005
quality ~				
control	-0.680	0.082	-8.262	0.000
known	0.175	0.047	3.717	0.000
control ~				
underst	-0.361	0.070	-5.146	0.000
known	-0.228	0.048	-4.737	0.000
underst $\sim$				
known	0.031	0.045	0.683	0.495
mins	0.219	0.091	2.404	0.016
citem	0.382	0.172	2.217	0.027
cfriend	0.541	0.178	3.046	0.002
cgraph	0.432	0.147	2.950	0.003
mins ~				
known	0.013	0.039	0.325	0.745
citem	-0.184	0.097	-1.898	0.058
cfriend	-0.396	0.128	-3.104	0.002
cgraph	0.279	0.095	2.933	0.003
known ~				
citem	0.255	0.364	0.701	0.483
cfriend	0.602	0.355	1.696	0.090
cgraph	0.750	0.303	2.477	0.013



#### Average rating

- Outcome of recommendation quality (and maybe number of known recommendations)
- May influence satisfaction?



```
model <- 'satisf =~ s1+s2+s3+s4+s5+s6+s7
quality =~ q1+q2+q3+q4+q5+q6
control =~ c1+c2+c3+c4
underst =~ u2+u4+u5
satisf ~ rating+quality+control+known
rating ~
quality+control+underst+mins+known+citem+cfriend+cgraph
quality ~ control+known
control ~ underst+known
underst ~ mins+citem+cfriend+cgraph
mins ~ citem+cfriend+cgraph
known ~ cgraph'
```



Regressions:				
rating	-0.394	0.214	-1.841	0.066
quality	0.605	0.134	4.532	0.000
control	-0.885	0.125	-7.081	0.000
known	-0.108	0.050	-2.156	0.031
rating ~				
quality	0.354	0.039	9.090	0.000
control	-0.028	0.046	-0.614	0.539
underst	-0.050	0.041	-1.210	0.226
mins	-0.052	0.047	-1.109	0.267
known	0.048	0.021	2.252	0.024
citem	-0.183	0.089	-2.060	0.039
cfriend	0.054	0.101	0.533	0.594
cgraph	-0.054	0.077	-0.698	0.485



### We also measured:

- Music expertise (e1-e4)
- Trusting propensity (t1-t6)
- Familiarity with recommenders (f1-f6)



```
model <- 'satisf =~ s1+s2+s3+s4+s5+s6+s7
quality =~ q1+q2+q3+q4+q5+q6
control =~ c1+c2+c3+c4
underst =~ u2+u4+u5
expert =~ e1+e2+e3+e4
trust =~ t1+t2+t3+t4+t5+t6
fam =~ f1+f2+f3+f4+f5+f6'</pre>
```

fit <- sem(model,data=twq,ordered=names(twq[9:47]),std.lv=T)</pre>

summary(fit, rsquare=T)



## Trimming:

- Remove t5 and t6 (low  $R^2$ )
- Remove t4 (low  $R^2$ )

Look at the modification indices:

There are a ton of things wrong with "familiarity"...



#### Items:

- I am familiar with online recommender systems.
- I have occasionally followed the advice of a recommender system.
- I have heard of systems similar to TasteWeights.
- I have never used anything like Taste Weights before.
- I regularly use systems similar to TasteWeights.
- Using Taste Weights was a completely new experience for me.



Might be two factors!

f1-f2: familiarity with recommender systems (fam)

f3-f6: familiarity with system like TasteWeights (famtw)

We eventually decided to only keep fam (remove famtw)

```
Model:
  model <- 'satisf =~ s1+s2+s3+s4+s5+s6+s7
  quality =~ q1+q2+q3+q4+q5+q6
  control =~ c1+c2+c3+c4
  underst =~ u2+u4+u5
  expert =~ e1+e2+e3+e4
  trust =~ t1+t2+t3
  fam =~ f1+f2'
```



```
model <- 'satisf =~ s1+s2+s3+s4+s5+s6+s7
quality =~ q1+q2+q3+q4+q5+q6
control =~ c1+c2+c3+c4
underst =~ u2+u4+u5
expert =~ e1+e2+e3+e4
trust =~ t1+t2+t3
fam =~ f1+f2</pre>
```

```
satisf ~ quality+control+known+expert+trust+fam
rating ~ quality+known+expert+trust+fam
quality ~ control+known+expert+trust+fam
control ~ underst+known+expert+trust+fam
underst ~ expert+trust+fam+mins+citem+cfriend+cgraph
mins ~ expert+trust+fam+citem+cfriend+cgraph
known ~ expert+trust+fam+cgraph'
```



Regressions:				
satisf ~				
quality	0.425	0.085	4.983	0.000
control	-0.898	0.136	-6.615	0.000
known	-0.159	0.065	-2.426	0.015
expert	0.131	0.089	1.470	0.141
trust	0.140	0.078	1.794	0.073
fam	0.132	0.098	1.339	0.180
rating ~				
quality	0.324	0.031	10.540	0.000
known	0.071	0.023	3.146	0.002
expert	0.006	0.034	0.163	0.870
trust	0.045	0.033	1.372	0.170
fam	-0.020	0.035	-0.573	0.567
quality ~				
control	-0.719	0.087	-8.287	0.000
known	0.143	0.049	2.895	0.004
expert	0.212	0.081	2.633	0.008
trust	0.008	0.080	0.102	0.919
fam	0.197	0.084	2.349	0.019
control ~				
underst	-0.410	0.077	-5.315	0.000
known	-0.280	0.050	-5.554	0.000
expert	0.249	0.081	3.062	0.002
trust	-0.110	0.075	-1.479	0.139
fam	0.281	0.082	3.420	0.001



underst $\sim$				
expert	0.072	0.079	0.908	0.364
trust	0.009	0.068	0.131	0.896
fam	0.167	0.073	2.307	0.021
mins	0.211	0.110	1.911	0.056
citem	0.421	0.208	2.024	0.043
cfriend	0.656	0.205	3.195	0.001
cgraph	0.456	0.147	3.108	0.002
mins $\sim$				
expert	0.027	0.048	0.577	0.564
trust	0.073	0.051	1.416	0.157
fam	-0.009	0.038	-0.233	0.816
citem	-0.181	0.097	-1.874	0.061
cfriend	-0.389	0.126	-3.093	0.002
cgraph	0.288	0.091	3.176	0.001
known ~				
expert	0.166	0.108	1.541	0.123
trust	-0.078	0.104	-0.751	0.453
fam	0.056	0.109	0.512	0.609
cgraph	0.667	0.298	2.236	0.025



Regressions:				
satisf ~				
quality	0.411	0.092	4.449	0.000
control	-0.956	0.148	-6.456	0.000
known	-0.153	0.064	-2.397	0.017
expert	0.205	0.100	2.055	0.040
trust	0.258	0.124	2.071	0.038
rating $\sim$				
quality	0.323	0.031	10.445	0.000
known	0.066	0.022	3.047	0.002
quality ~				
control	-0.770	0.094	-8.216	0.000
known	0.148	0.051	2.893	0.004
expert	0.374	0.094	3.987	0.000
control ~				
underst	-0.376	0.074	-5.087	0.000
known	-0.249	0.050	-5.037	0.000
expert	0.332	0.088	3.775	0.000



underst $\sim$				
fam	0.165	0.076	2.166	0.030
mins	0.230	0.114	2.025	0.043
citem	0.425	0.207	2.059	0.039
cfriend	0.665	0.205	3.247	0.001
cgraph	0.457	0.147	3.105	0.002
mins ~				
citem	-0.181	0.097	-1.874	0.061
cfriend	-0.389	0.126	-3.093	0.002
cgraph	0.288	0.091	3.176	0.001
known ~				
cgraph	0.698	0.305	2.291	0.022





Figure 3. The structural equation model for the data of the experiment. Significance levels: \*\*\* p < .001, \*\* p < .01, 'ns' p > .05.  $R^2$  is the proportion of variance explained by the model. Numbers on the arrows (and their thickness) represent the  $\beta$  coefficients (and standard error) of the effect. Factors are scaled to have an SD of 1.

## "It is the mark of a truly intelligent person to be moved by statistics."

## 

George Bernard Shaw